TWGHs Lo Kon Ting Memorial College

S2 Mathematics 2020 – 2021

STEM Worksheet --- Creating a Model of Pythagoras' Theorem

Worksheet 2

Name: ______________________ Class: _____ (   ) Group: ___ Date: ______________

Pythagoras' Theorem

In $\triangle ABC$, if $\angle C = 90^\circ$, then $a^2 + b^2 = c^2$.

Proof of Pythagoras' Theorem by Rearrangement

- Figure 2.1(a) shows a square with 4 *right-angled* triangles and 2 squares of side a and b respectively.
- Figure 2.1(b) shows a square with 4 *right-angled* triangles and a square of side c.

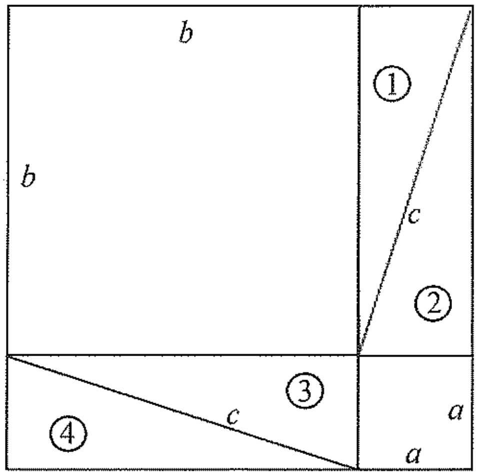

Figure 2.1(a)

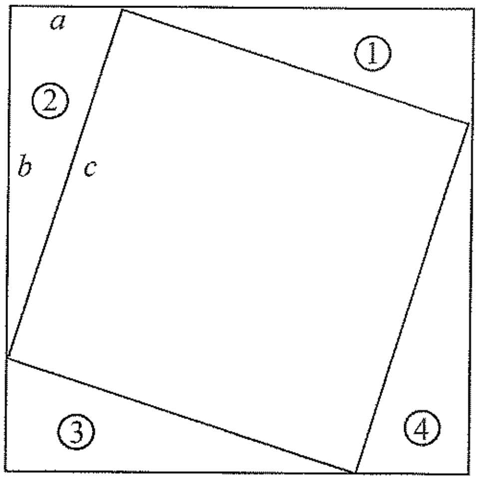

Figure 2.1(b)

Based on the figures above, how can we prove that $a^2 + b^2 = c^2$?

The area of Figure 2.1(a) = The area of Figure 2.1(b)

$= (a+b)^2$

$\because$ The triangles ①, ②, ③, ④ are the same in both figures.

By removing the 4 triangles, we have:

$a^2 + b^2 = c^2$

Proof of Pythagoras' Theorem by Similar Triangles

Figure 2.2 shows 3 right-angled triangles (namely ΔABC, ΔADB and ΔBDC).

Suppose $\angle ACB = x$.

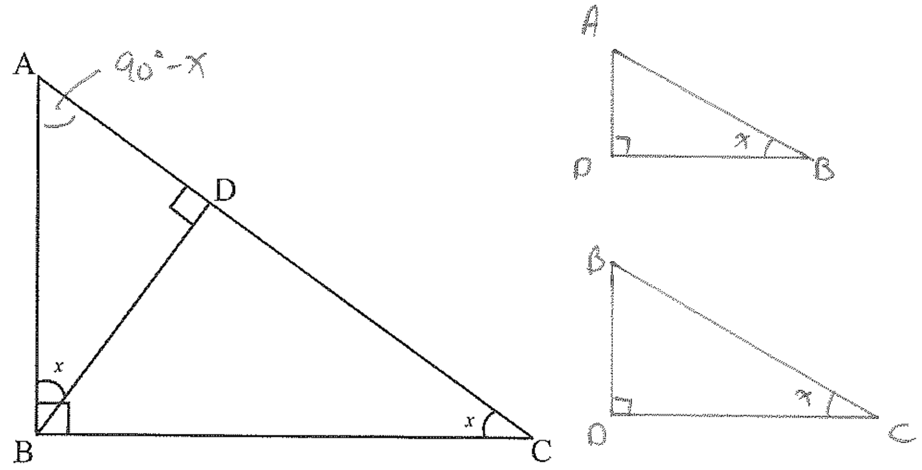

Figure 2.2

(a) Show that $\angle ABD = x$.

Consider ΔABC,

$$90° + x + \angle BAC = 180° \quad (\angle \text{ sum of } \Delta)$$

$$\angle BAC = 90° - x$$

Consider ΔABD,

$$90° - x + 90° + \angle ABD = 180° \quad (\angle \text{ sum of } \Delta)$$

$$\angle ABD = x.$$

(b) Show that $\Delta ABC \sim \Delta ADB$.

$\angle ABC = \angle ADB = 90°$ (given)

$\angle BAC = \angle DAB$ (common)

$\angle ACB = \angle ABD$ (proved in (a))

$\therefore \Delta ABC \sim \Delta ADB$ (A.A.A.)

(c) Also, it is given that $\Delta ABC \sim \Delta BDC$. Show that $AB^2 + BC^2 = AC^2$.

Consider $\Delta ABC \sim \Delta ADB$.

$$\frac{AC}{AB} = \frac{AB}{AD} \quad (\text{corr. sides}, \sim \Delta s) \Rightarrow AC \cdot AD = AB^2 \quad \ldots (1)$$

Consider $\Delta ABC \sim \Delta BDC$

$$\frac{AC}{BC} = \frac{BC}{DC} \quad (\text{corr sides}, \sim \Delta s) \Rightarrow AC \cdot DC = BC^2 \quad \ldots (2)$$

$(1) + (2)$:

$$AB^2 + BC^2 = AC \cdot AD + AC \times DC$$

$$= AC\,(AD + DC)$$

$$= AC \cdot AC$$

$$= AC^2$$