

Name: _____ Class: _____ () Group: _____ Date: _____

Pythagoras' Theorem

In $\triangle ABC$, if $\angle C = 90^\circ$, then $a^2 + b^2 = c^2$.

Proof of Pythagoras' Theorem by Areas

Figure 1.1 shows a big square of side c . It consists of:

- 4 identical right-angled triangles;
- and 1 small square of side $(b - a)$.

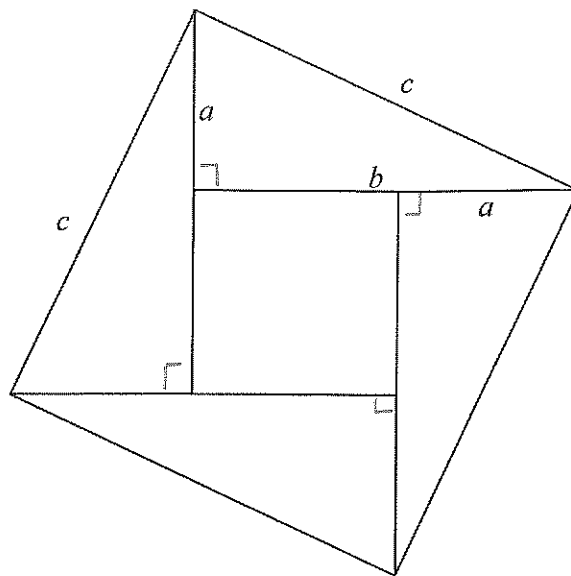


Figure 1.1

Based on the figure above, how can we prove that $a^2 + b^2 = c^2$?

(Hint: Consider the area of the big square, 4 triangles and the small square.)

$$\text{Area of the big square} = c^2$$

$$\text{Area of the 4 triangles} = 4 \times \frac{ab}{2} = 2ab$$

$$\text{Area of the small square} = (b-a)^2 = b^2 - 2ab + a^2$$

$$\therefore \text{Area of the big square} = \text{Area of the 4 triangles} + \text{Area of the small square}$$

$$\therefore c^2 = 2ab + b^2 - 2ab + a^2$$

$$c^2 = a^2 + b^2$$

□

Proof of Pythagoras' Theorem by Rearrangement (重排) and Areas

- Figure 1.2(a) is the same as Figure 1.1 on the previous page.
- After rearrangement, we have Figure 1.2(b).

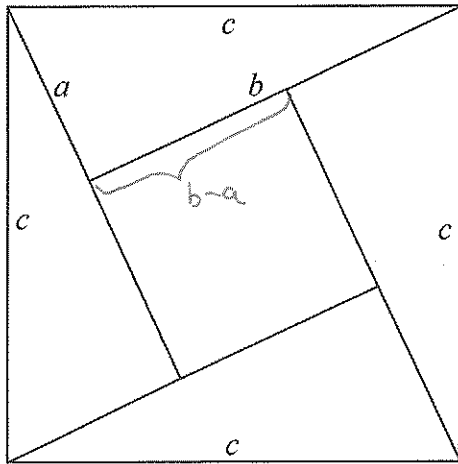


Figure 1.2(a)

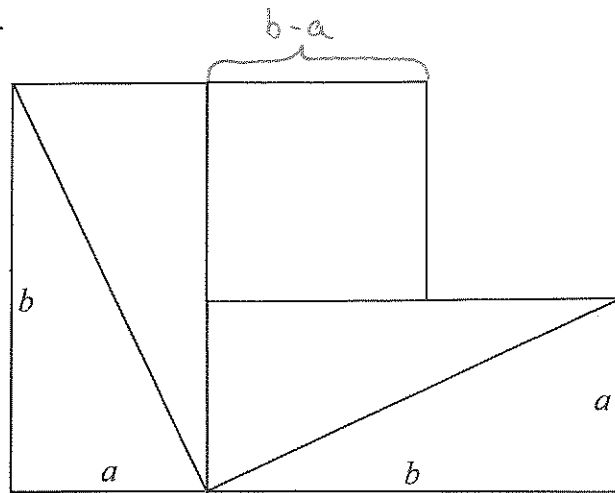


Figure 1.2(b)

Based on the figures above, how can we prove that $a^2 + b^2 = c^2$?

After rearrangement, the areas of Figure 1.2(a) and Figure 1.2(b) are the same.

$$\therefore c^2 = a \cdot b + a \cdot b + (b-a)^2$$

$$c^2 = 2ab + b^2 - 2ab + a^2$$

$$c^2 = a^2 + b^2$$

□